08-03-2002

Toepassingen van de KwantumFysica

> Solve each of the 4 problems on a separate sheet of paper
> Write your name on each sheet
> Mobile phones should be switched off during the exam

1. Binding Energies, Configurations, Terms and States

Consider a He atom in the groundstate. The total binding energy of both electrons together is 78.9 eV. (NB. The binding energy of the 1s-electron in hydrogen is 13.6 eV.)

4 pts a) Determine the effective nuclear charge "seen" by each of the electrons.

4 pts b) What is the minimum energy, necessary to singly ionize the He atom? (use the fact, that the energy necessary to subsequently remove the second electron can be calculated exactly)

Consider now the case that both electrons are in the n=2 shell

4 pts c) Which configurations are possible?

4 pts d) Which of the configurations is most strongly bound? Give arguments for your answer.

5 pts e) Give the possible terms and states for each configuration.

4 pts f) In a schematic drawing indicate the binding energies of all states for which the *l* quantum numbers of both electrons are equal.

2. Hyperfine splitting and Lamb-dip maxima

Consider an atom which in its $^3P_2$ excited state exhibits a hyperfine splitting of 30, 40, 50 and 60 MHz between subsequent hyperfine states.

4 pts a) Determine the hyperfine constant a [in MHz].

3 pts b) Give the values of *F*.

3 pts c) What is the nuclear spin *I* of the atom.

The ground state of the atom is a $^3S_1$ state.

3 pts d) Give the values of *F* of this ground state.

A Doppler-free saturation spectroscopy experiment is performed on this atom.

4 pts e) Briefly describe and/or sketch Doppler-free saturation spectroscopy.

4 pts f) Calculate the frequencies at which Lamb-dip maxima occur, for the case that in the ground state only the highest F level is active. Take the frequency of the transition between the highest F level in the ground state and the highest F level in the excited state (6×10^{14} Hz) as zero point.

4 pts g) The different peaks correspond to interactions with different velocity classes of atoms. What is the velocity of the atoms (in the direction of the laser) contributing to the peak which is most shifted from the zero point.

3. Atoms in magnetic fields

Consider the ^2S ground state term and the ^2P excited term in for example an alkali atom with no nuclear spin. The magnetic field B=0.

2 pts a) Which J states contribute to these ^2S and ^2P terms.

2 pts b) How many lines will be observed in a spectrum.

Consider a weak magnetic field.

3 pts c) Sketch the splitting of the J states induced by the magnetic field and give the relevant quantum numbers.

3 pts d) Calculate the splitting in units of $\mu_B B$.

3 pts e) How many lines will be observed in a spectrum.

Consider a strong magnetic field.

3 pts f) Sketch the splitting of the states induced by the magnetic field and give the relevant quantum numbers.

3 pts g) Calculate the splitting in units of $\mu_B B$.

3 pts h) How many lines will be observed in a spectrum.

3 pts i) Describe and/or sketch the precessional motion of the relevant angular momenta vectors with respect to one another and the magnetic field both for the weak and strong field cases.

$$g_J = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

4. Perturbation theory

Consider a rigid rotor-like system of which the Hamiltonian is

$$H = \frac{-\hbar^2}{MR^2}\frac{\partial^2}{\partial\varphi^2}$$

And the eigenstates u_n are $u_n = Ce^{in\varphi}$ with $C = (2\pi)^{-1/2}$

5 pts a) Give the time-independent Schrödinger equation for this system and calculate the eigen energies.

Now the system is perturbed by a small additional potential of $A\cos 2\varphi$. In the following consider the two degenerate states with n= -1 and 1.

20 pts b) Calculate the energies and states of this perturbed system (If you have the impression that you may spend too much time on the calculation describe briefly the procedure how to calculate the energies and states of this perturbed system).